ULTRASOUND MEASURES BOTH BONE MASS AND ARCHITECTURE: THEORETICAL RESULTS FOR AN IDEALIZED MODEL OF TRABECULAR BONE

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INTRODUCTION: Ultrasound is currently being investigated as a means for non-invasively assessing bone. It has been suggested since ultrasound is a mechanical wave that it may be able to evaluate additional factors related to bone quality in comparison to bone density alone [1]. Although ultrasonic methods appear promising in terms of their ability to assess both bone density and fracture risk, there is still insufficient understanding regarding the relationship between ultrasonic measurements and trabecular bone quality. The objective of this study was to determine theoretical relationships existing between ultrasonic velocity and bone density and architecture in an idealized model of trabecular bone, as a means for further clarifying the ways in which ultrasound interacts with bone.

MATERIALS AND METHODS: An idealized model of trabecular bone was developed consisting of a three-dimensional arrangement of identical beams or rods arranged as a cubic crystal. Each rod had a square cross section of area d^2 and length ℓ. Junctions of the rods were formed at 90° angles and considered rigid. The beams themselves were considered to be homogenous and generalized Hooke's law validity assumed. The rods were isotropic with a Young's modulus, E, Poisson ratio, ν, and density, ρ_s, while the material which filled the "pores" of the three-dimensional structure was assumed to offer essentially no elastic resistance, *e.g.*, air. The coefficients of the compliance matrix, **S**, relating the strains to the stresses on the structure, were found using the thin beam approximation and by calculating the strains due to longitudinal and shear stresses independently [2]. The velocity of a longitudinal ultrasound wave was then computed based on the above compliance matrix according to the Christoffel relations [3], assuming an infinite propagation medium and a wavelength much larger than the individual cubic beam elements making up the overall structure. The ultrasonic velocity, v, was computed for a range of volume fractions (0.01-0.40) and propagation angles (0°-52°).

RESULTS: The non-zero coefficients of the compliance matrix of the trabecular model are given by

$$s_{11} = s_{22} = s_{33} = 1/(Eb^2)$$

$$s_{44} = s_{55} = s_{66} = 1/(Eb^4)$$

$$s_{12} = s_{21} = s_{13} = s_{31} = s_{23} = s_{32} = \nu/(Eb)$$

where $b = d/\ell$. The velocity of the ultrasonic wave is given by

$$v = \sqrt{\frac{E}{\rho_s}}\, b \sqrt{\frac{\lambda}{1-\phi}}$$

where ϕ is the porosity of the structure, and λ is the maximum eigenvalue associated with the solution of the Christoffel equations, and is dependent on the compliance constants s_{ij} as well as the porosity and angle at which the wave is propagating [3]. For this cubic beam model, the porosity is given by $\phi = (1-b)^2(1+2b)$. Fig. 1 displays the ultrasonic velocity *vs.* solid volume fraction $(1-\phi)$, for several propagation directions.

DISCUSSION AND CONCLUSION: A simple model representing an idealized trabecular bone structure was used to analytically evaluate the ultrasonic propagation velocity. The results shown in Fig. 1 demonstrate that the observed velocity is a function not only of density but also of the particular orientation at which the ultrasonic wave is assumed to be propagating. This dependence of velocity is likely due to the relative amount of bending *vs.* pure compression loading present in the given propagation direction. The velocity is seen to experience about the same order of changes from architectural differences as with density variations. The above observations also demonstrate that observed velocity values cannot necessarily be used to uniquely determine the density, unless architectural aspects are taken into account. The results also help to illuminate the often posed question, "What does ultrasound measure?" [1,4]. A reasonable answer is that ultrasound measures aspects of both density *and* structure. The model and analysis also can be extended to more realistic situations, such as asymmetric cubic structures and fluid-filled ones as well. This and subsequent studies should lead to a better understanding of the way in which ultrasound interacts with bone, towards the ultimate goal of making ultrasonic techniques useful for clinical bone assessment.

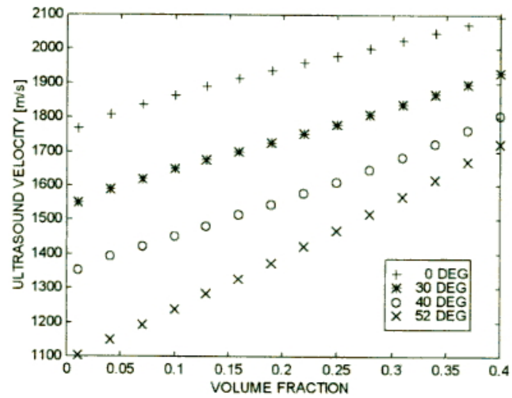

Fig. 1. Ultrasonic velocity for the idealized trabecular structure.

REFERENCES: [1] Kaufman JJ, Einhorn TE *J Bone Min Res*, *8*:517, 1993. [2] Bianco B, *Technical Report #BB001A*, CyberLogic, Inc., 1994. [3] Love AEH *A Treatise on the Mathematical Theory of Elasticity*, NY:Dover, 1944. [4] Schott *et al. J Bone Min Res*, *10*:243, 1995.

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