# Diffraction Effects in Insertion Mode Estimation of Ultrasonic Group Velocity

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***Abstract*— We describe diffraction effects in ultrasonic group velocity estimation using an insertion technique. We characterize the estimation error produced by diffraction as a function of distance and nominal velocity values. A new method termed Group Velocity Diffraction Correction (GVDC) which corrects for the diffraction effect is presented. Experimental validation of the technique is also presented using measurements made with both 1 MHz and 500 kHz ultrasonic transducer pairs. The results demonstrate that diffraction effects on ultrasonic group velocity estimation are usually small, and may often be neglected. Significant improvement, up to about 50%, in the accuracy of the group velocity estimate can however be obtained using the method described here in those cases in which higher degrees of accuracy are required.**

## I. Introduction

The estimation of ultrasonic velocity is an important area of study. Its significance lies primarily with the need for determining nondestructively the physical state of various materials, and for providing a means for recognizing and classifying materials into specific categories. These categories may include for example different degrees of material strength, or degrees of porosity. Many applications have been reported from both the biomedical and industrial nondestructive evaluation (NDE) communities. These range from classification of disease pathologies [1]–[5] to industrial inspection applications [6]–[8]. Various techniques for velocity estimation have been reported, from simple approaches using pulse-echo-overlap [9], to more complicated methods using spectral analysis and correlation. The latter approaches may be based on a variety of measuring conditions: transmission, substitution, and backscattering measurements [5].

Many factors affect the accuracy and precision of the ultrasonic velocity estimate, including for example temperature effects, digitization accuracy, and method of measurement [5]. Another factor influencing the velocity estimate is the effect of diffraction, that is, the changes with depth of the sound field produced by an ultrasound transducer, in comparison with planar wave propagation. Because of the finite size of the transducer, the acoustic beam spreads out into a complex and depth dependent field pattern, and this can produce range-dependent effects associated with ultrasonic measurements.

Most previous studies reported on the influence of diffraction on the measurement of ultrasonic attenuation, where its effect is usually more significant [10]–[18]. Early work on this subject was reported by Seki [10], in which he characterized the diffraction loss as a function of frequency and the distance from the transducer. It was observed that large errors, i.e., greater than 20%, in attenuation values could be produced by diffraction. Subsequent investigations led to the development of several diffraction correction techniques for attenuation estimation, primarily for the pulse-echo mode [11]–[18]. We reported recently [19] on a diffraction correction method for ultrasonic attenuation estimation using insertion mode measurements.

Much less work has been reported on diffraction effects in ultrasonic velocity estimation [20], [21]. Papadakis has presented some studies on phase correction methods to compensate for diffraction effects [22], [23]. For the most part, these results are applicable to phase velocity corrections. Less attention has been given to diffraction effects in insertion measurements of ultrasonic group velocity. This is because diffraction effects can usually be neglected in insertion methods when the respective velocities of ultrasound in the unknown and reference media are approximately equal. For example, when insertion measurements are carried out to identify the velocity of ultrasound in biological soft tissue, and the reference medium is water, diffraction effects are so small as to be essentially insignificant [5]. However, if the two media have significantly different velocities, i.e., more than a 50% difference, then diffraction can contribute to significant errors (greater than 1%) in velocity estimates, as we will show in the sequel. For example, velocity measurements in bone using an insertion technique with water as the reference medium can be significantly affected by diffraction since the velocity of ultrasound in bone is almost twice that in water [5].

In this paper, the influence of diffraction on the insertion estimation of the differential phase spectrum (i.e., ultrasonic group velocity) will be investigated and a method for its correction presented. The following section describes the problem in detail, including its experimental and theoretical aspects. Section III presents a method for assessing the effect of diffraction on group velocity estimates. We also describe a diffraction correction technique, which is termed Group Velocity Diffraction Correction (GVDC). The GVDC method corrects the measured differential phase spectrum using an

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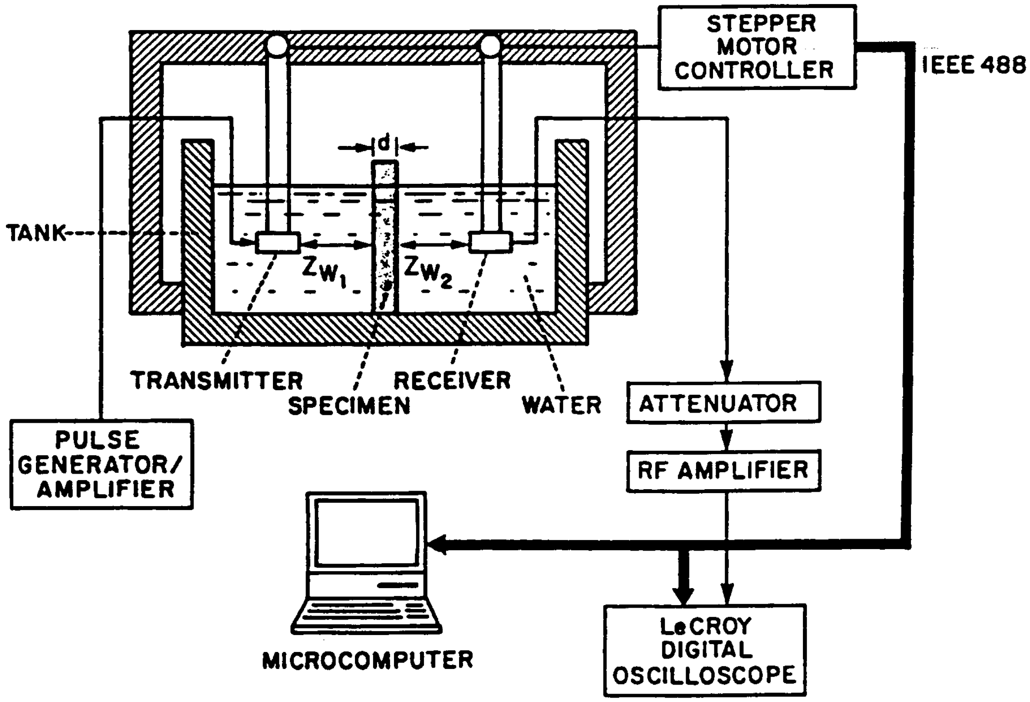



Fig. 1. Schematic of the experimental setup for insertion measurement of ultrasonic group velocity.

inverse diffraction transfer function obtained by numerical computer evaluation. Experimental validation of the diffraction correction technique appears in Section IV. We also include computer simulations which characterize diffraction effects in various media with several different velocities. A discussion and summary of the work concludes the paper.

## II. Description of Problem

### A. Experimental Set-Up

Fig. 1 exhibits a schematic of the experimental setup for the ultrasound measurements and one which we use in the analysis that follows. Two pairs of transducers are used, a set of 1 MHz broadband transducers (Panametrics V314) and a set of 500 kHz broadband transducers (Panametrics V318), all having a radius $a = 0.95$ cm. The pair of transducers are coaxially aligned in a tank containing degassed distilled water. The transmitting transducer is excited with a 300 volt 0.5 microsecond pulse produced with an Analogic Data Precision arbitrary function generator (Model 2020) and an ENI (Model #240L) radio frequency power amplifier. The acoustic pulse propagates in the water, through the specimen for which the group velocity is desired, and through the water again until it is received at the receiving transducer. This transducer then converts the acoustic pulse into an electrical waveform which is sampled at a rate of 20 MHz and digitized by a storage oscilloscope (LeCroy 9400). The data is then uploaded to a microcomputer for storage and off-line analysis. A stepper motor controller enables three-dimensional movement of the transducers in steps of 0.1 mm. Specimen thickness and transducer-specimen separations are chosen such that only the pulse transmitted directly through the specimen is recorded without interference from multiple reflections. In particular, the round trip travel times of the acoustic pulses associated with the sample are long enough to ensure that the first multiple reflection arrives at the receiving transducer after the primary waveform has decayed to the noise level. The specimen is aligned parallel to the transducer faces by maximizing the amplitude of the near surface reflection by operating the transmitter in pulse-echo mode.

For the empirical data, measurements of group velocity are obtained from a 2.4 cm thick block of polymethylmethacrylate (PMMA). An insertion method as described in the following paragraphs is used to measure the ultrasonic group velocity in the sample. The velocity of ultrasound in polymethylmethacrylate is almost twice that in water, namely 2770 $\mathrm{ms}^{-1}$ [5], and thus it has the potential for exhibiting diffraction effects on the group velocity estimate.

### B. Analytic Description

Let $v(t)$ denote the input electrical signal to the transducer. The spectrum of the received waveform after it has propagated through the water-PMMA-water complex is given by $Y_e(f, z)$:

$$Y_e(f,z) = H_w^1(f)H_s(f)H_w^2(f)H_d^e(f,z)H_T(f)H_R(f)V(f) \tag{1}$$

where $f$ is the frequency in cycles per second [Hz] and $z$ is the distance between the two transducers. Here, $V(f)$ is the Fourier transform of the input signal, $v(t), H_T(f)$ and $H_R(f)$are the transfer functions of the transmitting and receiving transducers loaded with water, respectively, and $H_w^1(f), H_w^2(f)$ and $H_s(f)$ are the acoustic transfer functions of an incident planar wave in the water path between the transmitting transducer and sample, in the water path between the sample and receiving transducer, and in the sample itself, respectively. All diffraction effects are incorporated in $H_d^e(f,z)$, which characterizes the effect of diffraction on the ultrasound pulse for the water-sample-water propagation path, and is a function of the transducer separation distance, $z$, and transducer radius, $a$, as well as the acoustic properties of the water and specimen, respectively.

Similarly, $Y_r(f,z)$ is the Fourier transform of the ultrasound signal which has propagated through the water path only, i.e., without the sample present, and is given by

$$Y_r(f,z) = H_w^1(f)H_w^s(f)H_w^2(f)H_d^r(f,z)H_T(f)H_R(f)V(f). \tag{2}$$

In (2), $H_d^r(f,z)$ characterizes the effect of diffraction on the received acoustic waveform for the water only path, and $H_w^s(f)$ is the acoustic transfer function of an incident planar wave in the water which is displaced by insertion of the sample into the reference media. Note that in (1) and (2) we have neglected any acoustic transmission and reflection coefficients, for example as between the water and sample, since they are essentially frequency independent [5], and therefore will not affect the analysis to follow. In practice, the acoustic transmission and reflection coefficients provide constant gain factors which will not affect the phase estimates used in the group velocity estimation procedure. In the same spirit we have neglected any multiple reflections and assume that time gating of an appropriate nature can be used to effectively isolate the primary incident acoustic waveform.

An estimate, $\hat{H}_s(f,z)$, of the sample's acoustic transfer function, $H_s(f)$, is obtained by dividing (1) by (2) and multiplying the result by $H_w^s(f)$:

$$\hat{H}_s(f,z) = \frac{Y_e(f,z)}{Y_r(f,z)}H_w^s(f) = H_s(t)\frac{H_d^e(f,z)}{H_d^r(f,z)} \tag{3}$$

In (3), it is assumed that $H_w^s(f)$ is known and is equal to $e^{-j2\pi fd/V_w}$, where $V_w$ is the ultrasonic velocity in the reference or in this case water media, and $d$ is the sample thickness. The estimated sample transfer function is seen to be modified by the effect of diffraction. This may be characterized by the *diffraction transfer function (DTF)*, $H_d(f,z)$, defined by

$$H_d(f,z) \triangleq \frac{H_d^e(f,z)}{H_d^r(f,z)} \tag{4}$$

and thus

$$\hat{H}_s(f,z) = H_s(f)H_d(f,z). \tag{5}$$

Let the sample's transfer function be written as

$$H_s(f) = A_s(f)e^{-j\phi_s(f)} \tag{6}$$

where $\phi_s(f)$ is the phase of the specimen's acoustic transfer function measured in radians, and $A_s(f)$ is its magnitude. The phase function can be expressed in terms of the specific phase function, $\beta_s(f)$, as

$$\phi_s(f) = \beta_s(f)d \tag{7}$$

where $d$ is the sample thickness. The group velocity is defined as

$$v_g(f) \triangleq 2\pi\left(\frac{d\beta_s(f)}{df}\right)^{-1}. \tag{8}$$

Neglecting the effect of diffraction, the group velocity estimate, $\hat{v}_g$, can be evaluated as

$$\hat{v}_g(f,z) = -2\pi d\left[\frac{d}{df}\arg\left(\frac{Y_e(f,z)}{Y_r(f,z)}H_w^s(f)\right)\right]^{-1}. \tag{9}$$

In (9) and (10), we assume that the arg function produces a continuous function of frequency $f$, or that the phase is appropriately *unwrapped* [24]. As may be seen, the estimate of the group velocity, $\hat{v}_g$, as defined by (9) is a function of the distance $z$ at which the insertion measurement is made. The actual value for the group velocity, $v_g$, is given by

$$v_g(f) = -2\pi d\left[\frac{d}{df}\arg\left(\frac{Y_e(f,z)}{Y_r(f,z)}\frac{H_w^s(f)}{H_d(f,z)}\right)\right]^{-1}. \tag{10}$$

The error, $e$, associated with using (9) instead of (10), i.e., $e = \hat{v}_g - v_g$, is due to diffraction of the ultrasound wave. As will be shown later, neglecting this effect can result in errors in the group velocity estimate. Note that we have neglected entirely the effect of measurement noise on the group velocity estimates, as this does not effect the analysis to follow.

## III. Group Velocity Diffraction Correction

### A. Basic Diffraction Theory

Acoustic waves emitted by a transducer into a specimen are not confined to a region contained within the geometrical shadow of the transducer and not perpendicular to its emitting surface. Because of the transducer's finite size, the ultrasonic wave spreads out into a diffraction field, a phenomenon that can introduce errors in both attenuation and velocity measurements. The diffraction effect is related to the ratio of source size to acoustic wavelength and thus is especially important for low frequencies and small transducers. It should also be pointed out that a similar diffraction phenomenon occurs for the acoustic wave impinging on the receiver. In general, there is also a further contribution to diffraction due to the mismatch between the sample and reference media, and to the sample's finite size.

Investigations of the effect of diffraction on attenuation and velocity measurements have been made by a number of authors in the case of circular ultrasound transducers. The transducer is usually treated as a finite piston source in an infinite rigid baffle radiation into a semi-infinite medium. The acoustic field is found at each point in the propagation medium and an integration is performed over a specified area, usually concentrically located with respect to the transducer source.

In the following analysis, we follow the approach of Kino [25]. Assume a transducer of radius $a$ on the $z = 0$ plane. The velocity potential $\Phi(x, y, z)$ can be expressed as

$$\Phi(x,y,z) = -\frac{1}{2\pi}\int_s u_z(x',y',0)\frac{e^{-jkR}}{R}dS' \tag{11}$$

where $R$ is the distance between the source point and the field point $(x, y, z), u_z(x', y', 0)$ is the displacement of the transducer surface in the normal or $z$-direction at location $x = x'$, $y = y'$, and $z = 0$, and $dS'$ is a differential element on the surface of the transducer. Using the Hankel transform, an expression for the acoustic displacement $\mu_z(r, z)$ in cylindrical coordinates $(r, \theta, z)$ can be found and expressed in terms of the parameters $k_r$, and $k_z$, the radial and axial spatial frequencies, respectively [25]:

$$\mu_z(r,z) = a\mu_0\int_0^\infty J_1(k_r a)J_0(k_r r)e^{-jk_z z}dk_r \tag{12}$$

and

$$k_r^2 + k_z^2 = k^2. \tag{13}$$

Here $J_1(\bullet)$ and $J_0(\bullet)$ are the first and zeroth order Bessel functions, respectively, $k = 2\pi/\lambda$, and $\lambda$ is the ultrasound wavelength. The response of a receiving transducer of radius $a$ located a distance $z$ from the transmitting transducer is proportional to the average displacement over its surface, i.e., $\bar{\mu}_z(z)$, where

$$\bar{\mu}_z(z) = \frac{2}{a^2}\int_0^a \mu_z(r,z)rdr. \tag{14}$$

Using (12) and the Fresnel approximation, i.e.,

$$k_z = \sqrt{k^2 - k_r^2} \cong k - \frac{k_r^2}{2k} \tag{15}$$

the average displacement $\bar{\mu}_z(z)$ at a distance $z$, i.e., at the receiver, is given by [25]

$$\bar{\mu}_z(z) = 2\mu_0 e^{-jkz}\int_0^\infty \frac{J_1^2(Y)}{Y}e^{jY^2\frac{S}{4\pi}}dY. \tag{16}$$

Here $S = z\lambda/a^2$ is the Fresnel parameter and $\mu_0$ is the displacement at $z = 0$. Note that (16) includes a plane wave propagation term $e^{-jkz}$. Since this term has already been incorporated in (1) and (2) through the complex transfer functions $H_w^1(f), H_w^2(f), H_w^s(f)$ and $H_s(f)$, we define a *modified average displacement function*, $\bar{\mu}_z^*(z)$

$$\bar{\mu}_z^*(z) = 2\mu_0\int_0^\infty \frac{J_1^2(Y)}{Y}e^{Jy^2\frac{S}{4\pi}}dY \tag{17}$$

We use (17) to derive the diffraction correction technique below.

### B. Diffraction Correction

The GVDC technique numerically evaluates $H_d(f, z)$ and uses it to adjust the group velocity measurements. In particular, once $H_d(f, z)$ is known, it may be used in (10) to obtain the diffraction corrected group velocity.

In order to evaluate $H_d(f, z)$, we use (17) with appropriate parameter values. Specifically, values for the Fresnel parameter $S$ must be determined. Two cases must be considered. The first case deals with the values associated with the water-sample-water complex, and the second with the values associated with the water path only. In the following analysis, we assume that the Fresnel approximation (15) holds. Accordingly, the use of this approximation makes it straightforward to evaluate the result when the ultrasonic wave propagates through layered media; the total value of $S$ is found by adding the values of $S$ determined for each region, using the appropriate values of $z$ and $\lambda$ in these regions (p. 175 in [25]). Note that this analysis assumes also that the frequency dependence of the transmission and reflection coefficients associated with each of the layered media can be ignored, that incidence is primarily normal, and that mode conversion is not occurring. The latter two assumptions are closely related to the use of the Fresnel approximation itself (p. 173 in [25]).

In view of the above assumptions, the value for $S$ in the case of the water-sample-water path, namely $S = S_e$, may be obtained by adding together the individual contributions from each layer of the propagation medium, using the appropriate values of $z$ and $\lambda$ in these regions [25]

$$S_e = S_{w1} + S_s + S_{w2} \tag{18}$$

In (18), $S_{w1} = z_{w1}\lambda_w/a^2$ is the Fresnel parameter for the water layer between the transmitter and sample ("near water layer"); $S_s = d\lambda_s/a^2$ is the Fresnel parameter for the sample layer; and $S_{w2} = z_{w2}\lambda/a^2$ is the Fresnel parameter for the water layer between the sample and the receiver ("far water layer"). Here, $z_{w1}$, $d$, $z_{w2}$ are the thicknesses of the near water layer, sample, and far water layer, respectively, and $\lambda_w$ and $\lambda_s$ are the wavelengths of ultrasound in water and sample, respectively.

A similar but simpler evaluation can be used to determine the value of $S$ for the water only path, $S = S_r$. In this case, $S_r = z\lambda_w/a^2$. These parameters can then be used to evaluate $H_d^e(f, z)$. Note that the thickness of the sample affects the GVDC technique as much as the total water path $z$, which may be seen from (18).

The diffraction effect for the water-sample-water path which was characterized earlier by $H_d^e(f, z)$ in (1) can now be analytically described by

$$H_d^e(f,z) = \frac{\bar{\mu}_{ze}^*(z)}{\mu_0} = 2\int_0^\infty \frac{J_1^2(Y)}{Y}e^{jY^2\frac{S_e}{4\pi}}dY \tag{19}$$

where $\bar{\mu}_{ze}^*(z)$ is the modified average displacement function on a concentric receiver of radius $a$ located at a distance $z$ from the sound source for the water-sample-water complex. Similarly, the diffraction effect for the water path only, $H_d^r(f, z)$

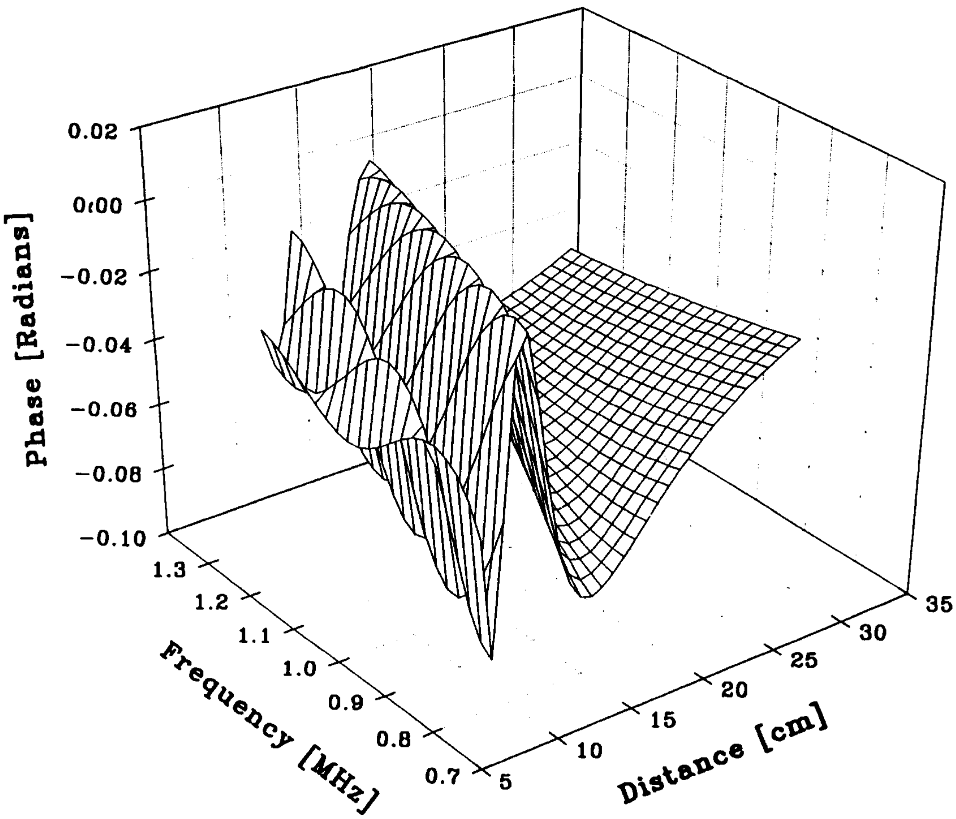



Fig. 2. The unwrapped phase, $\phi_d(f,z)$, of the diffraction transfer function, $H_d(f,z)$, for the frequency range 750 kHz—1.25 MHz, for distances of 5 cm to 34 cm, transducer radii of 0.95 cm, and sample thickness equal to 2.4 cm.

(2) can be described by

$$H_d^r(f,z) = \frac{\bar{\mu}^*_{zr}(z)}{\mu_0} = 2\int_0^\infty \frac{J_1^2(Y)}{Y} e^{jY^2\frac{S_r}{4\pi}}\, dY \tag{20}$$

where $\bar{\mu}^*_{zr}(z)$ is the modified average displacement function on a concentric receiver of radius $a$ located a distance $z$ from the source for the water only path. The diffraction transfer function (4) can then be derived using

$$H_d(f,z) = \frac{\left(\frac{\bar{\mu}^*_{zc}(z)}{\mu_0}\right)}{\left(\frac{\bar{\mu}^*_{zr}(z)}{\mu_0}\right)}. \tag{21}$$

Substituting (19) and (20) into (21), we obtain

$$H_d(f,z) = \frac{\int_0^\infty \frac{J_1^2(Y)}{Y} e^{jY^2\frac{S_c}{4\pi}}\, dY}{\int_0^\infty \frac{J_1^2(Y)}{Y} e^{jY^2\frac{S_r}{4\pi}}\, dY}. \tag{22}$$

$H_d(f,z)$ (22) can be closely approximated on a computer using standard numerical integration procedures.

## IV. Verification of the Analysis

In this section we present various simulation and experimental results which demonstrate the validity of the above analysis. We have previously presented data on the diffraction transfer function magnitude, $|H_d(f,z)|$ [19]. Here we present results for the phase of $H_d(f,z)$. Fig. 2 shows a plot of $\arg[H_d(f,z)]$, evaluated using (22). This simulation was generated over a frequency range of 750 kHz—1.25 MHz, for distances of 5 cm to 34 cm, and for transducers of radius 0.95 cm. The integrals in (22) were evaluated on a microcomputer with quadrature integration using MATLAB software (The Math Works Inc., South Natick, MA). In this and all subsequent simulations, the sample was chosen to have a thickness of 2.4 cm and the reference medium was water with a velocity of ultrasound of 1500 $\mathrm{ms}^{-1}$. In Fig. 2, the ultrasound velocity in the sample was assumed to be equal to that in polymethylmethacrylate, or 2770 $\mathrm{ms}^{-1}$. The plot shows that the largest diffraction effect is localized to the lower frequency values and shorter distances.

Next, we examine the effect of $H_d(f,z)$ on the ultrasonic group velocity estimate. In this simulation, the phase, $\phi_s(f)$, of the sample acoustic transfer function, $H_s(f)$, was modeled as

$$\phi_s(f) = \frac{2\pi f d}{v_s} \tag{23}$$

where $v_s$ is the velocity of ultrasound in the sample. $H_s(f)$ was then multiplied by $H_d(f,z)$ as in (5), to obtain the diffraction corrupted acoustic transfer function, $\hat{H}_s(f,z)$, of

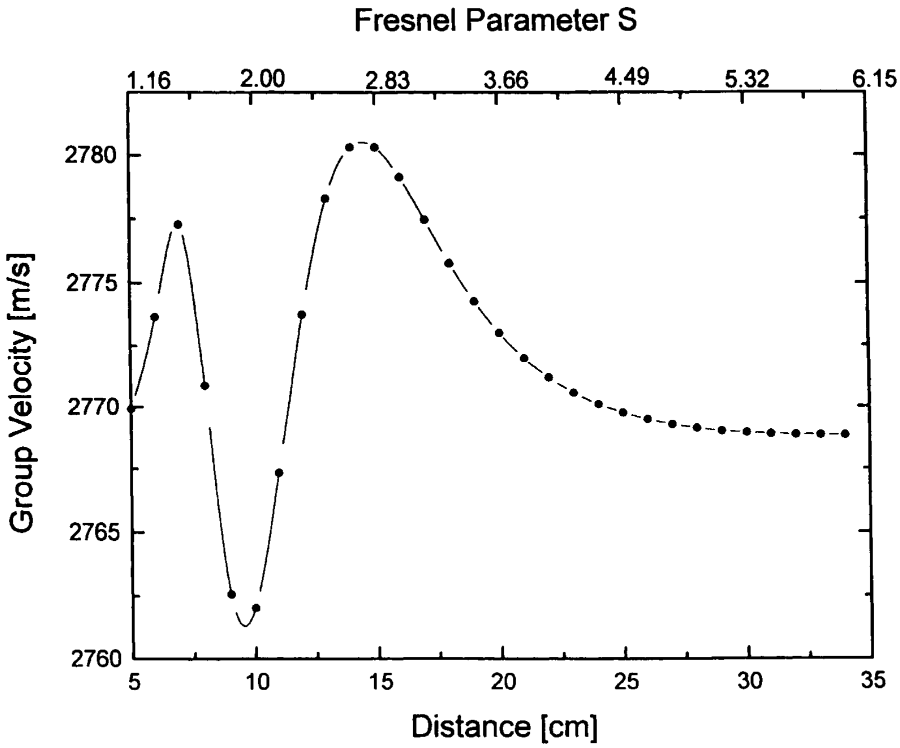



Fig. 3. Simulated ultrasonic group velocity $\hat{v}_g(f, z)$using a linear least squares curve fit over the frequency range 750 kHz—1.25 MHz, to the phase function estimate (see (9)).

the sample. We then carried out a linear least squares curve fit over the frequency range 750 kHz to 1.25 MHz to the unwrapped phase of $\hat{H}_s(f, z)$, and evaluated the ultrasonic group velocity, $\hat{v}_g(f, z)$, as in (9) as a function of distance $z$. The results of this simulation are shown in Fig. 3. As may be seen, the diffraction transfer function produces a small but definite effect on the group velocity estimate, with a maximum relative absolute error (MRAE), $e_m$, of almost 0.4%, and an average relative absolute error (ARAE), $e_a$, of 0.13%. The MRAE and ARAE are defined, respectively, by

$$e_m = \max_d \left\{ \left| \frac{e(d)}{v_g} \right| \right\}$$

and

$$e_a = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{e(d_i)}{v_g} \right|.$$

The results of the above simulation, that is, the values of the estimated group velocity, $\hat{v}_g(f, z)$, were compared to experimental results obtained using the set-up of Fig. 1, with the 1 MHz pair of ultrasonic transducers. Fig. 4(a) presents plots of the time domain voltage measurements of two typical acoustic signals through water and water-plastic-water paths, respectively. Fig. 4(b) presents the associated Fourier magnitude spectra of the two waveforms of Fig. 4(a). In Fig. 4(c) the magnitude of the acoustic transfer function (3) is shown, and in Fig. 4(d) the associated transfer function phase is plotted.

A linear least squares curve fit was performed over the same frequency range as above (750 kHz to 1.25 MHz) and over the distance range 5 cm to 34 cm, on the experimentally obtained phase data [see (9)]. Fig. 5 presents both the simulated and experimental diffraction corrupted group velocity estimates, respectively. Fig. 5(a) displays the experimental data for the 1 MHz transducer pair, and Fig. 5(b) displays the data for the 500 kHz transducer pair. As may be seen, there is excellent qualitative correspondence between the theoretical (i.e., computer simulated) and experimental group velocity values. Nevertheless, there are several points at which the simulated and experimental data display large differences from one another. This may be due in part to experimental errors in positioning of the ultrasonic transducers as well as errors in aligning the sample. In addition, the Fresnel approximation used in the analysis is only partially satisfied and there may also be problems associated with mode conversion due to nonnormal incidence.

We then applied the GVDC technique to the experimental data. The diffraction corrected and uncorrected group velocity estimates are shown in Figs. 6(a) and 6(b), for the 1 MHz and 500 kHz transducer pairs, respectively. Tables I and II present the performance of the GVDC scheme, for the 1 MHz and 500 kHz transducer pair data, respectively. There is significant improvement in the accuracy of the estimates, with the average absolute error improving from 0.15% to 0.075%, representing

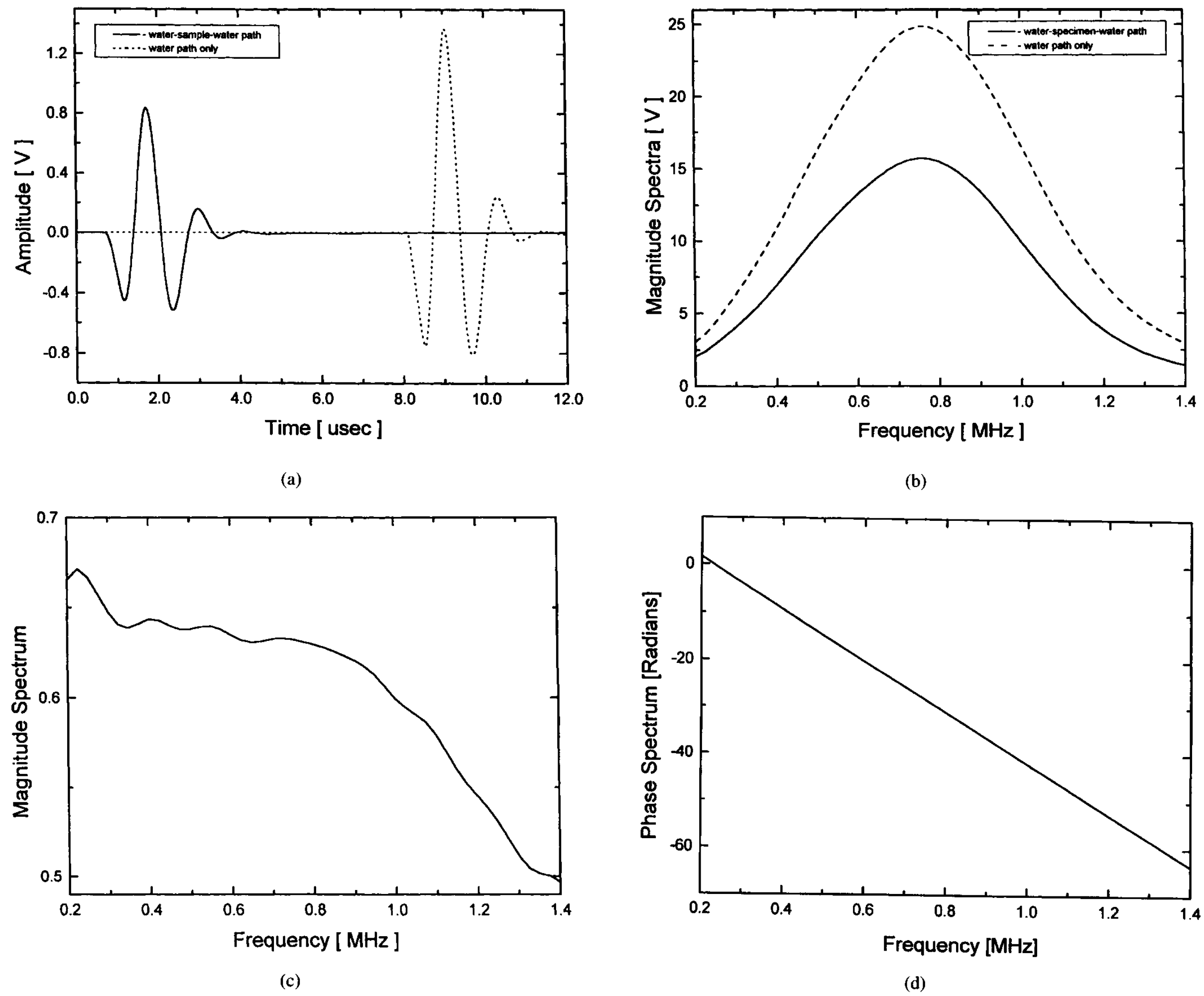


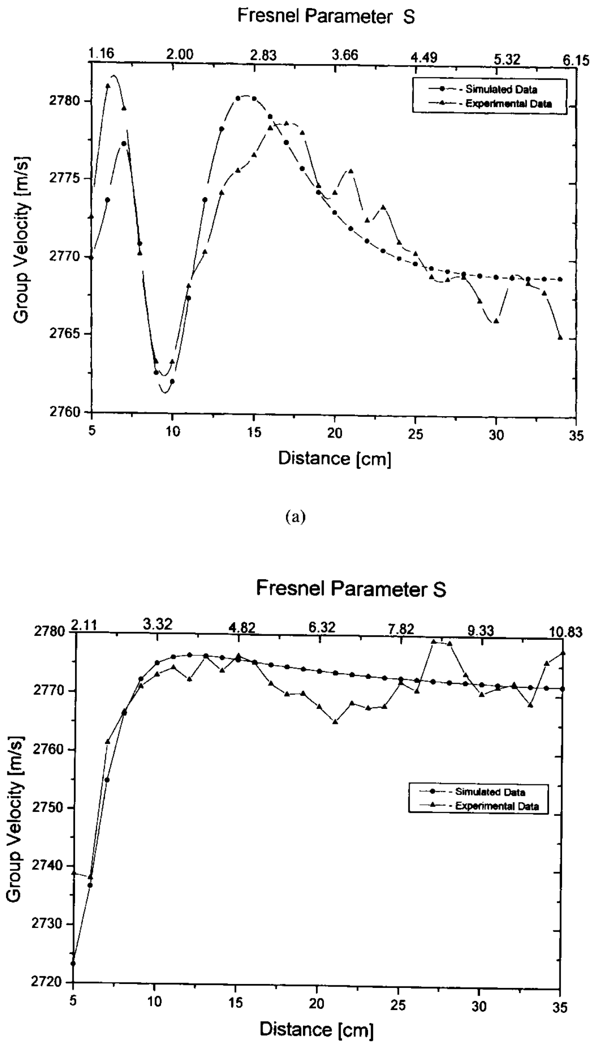

Fig. 4. (a) Ultrasonic time domain waveforms for a typical insertion measurement data set. The measurements were made on a 2.4 cm Lucite block, with a transducer separation of 10 cm. (b) Fourier amplitude spectra of the ultrasonic time domain waveforms in Fig. 4(a). The magnitude of the transfer function estimate of (3), using the acoustic waveforms of Fig. 4(a). (d) The phase of the transfer function estimate of (3), using the acoustic waveforms of Fig. 4(a).

an improvement of over 40% for the 1 MHz transducer pair. For the 500 kHz transducer pair, the maximum absolute percent error decreases from 1.2% to 0.58%, representing an improvement of about 52%. The RMS value of the velocity error improves (i.e., decreases) by about 50 %, for both the 1 MHz and 500 kHz transducer pairs. As stated in the preceding paragraph, various factors can contribute to the actual errors in the velocity estimates produced by the GVDC technique. In addition to those already mentioned, it should be pointed out that phase estimation of the acoustic signals is subject potentially to large errors, and these phase errors will thus affect the accuracy and precision of the estimates obtained.

We also calculated the effect of diffraction for several different sample velocity values. Figs. 7(a)–7(c) present the diffraction corrupted group velocity estimates for $v_s$ = 1000 ms$^{-1}$, 1600 ms$^{-1}$ and 6000 ms$^{-1}$, respectively. In all cases, the reference medium was assumed to be water with a velocity of $v_w$ = 1500 ms$^{-1}$. Table III provides the average absolute and maximum percent errors for all of the sample velocities evaluated here. As may be seen, the errors are all significantly below 1%, except for the sample velocity corresponding to steel, $v_s$ = 6000 ms$^{-1}$, in which the maximum percent error is 2.3%.

## V. Concluding Discussion

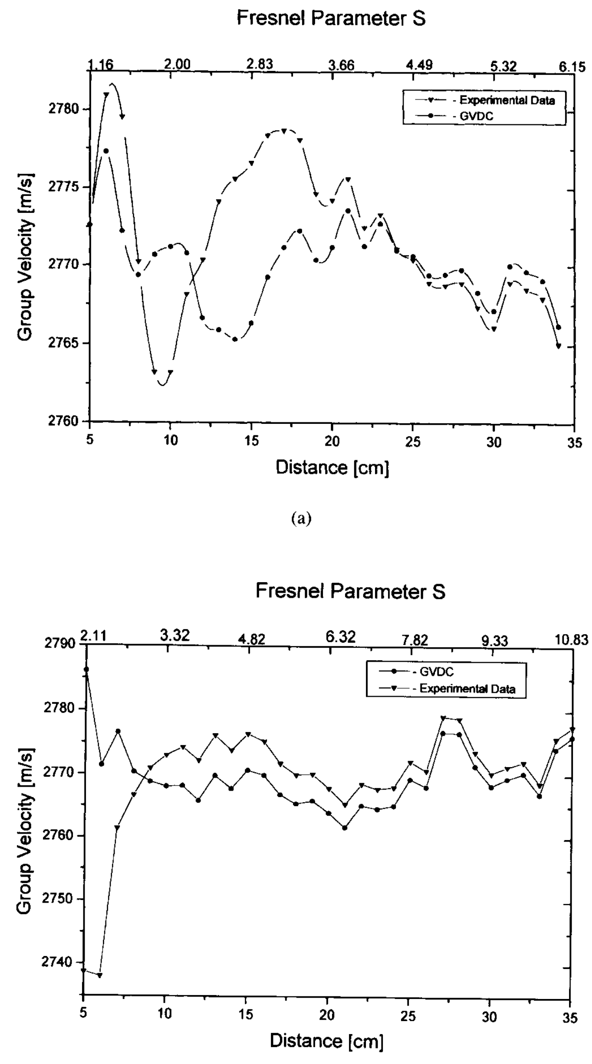
Most previous studies on diffraction addressed pulse-echo type measurements and its effect on ultrasonic attenuation estimation. We studied here the effect of diffraction on group velocity in an insertion mode measurement technique. We demonstrated that the error in group velocity is relatively small, and in most circumstances largely negligible. In cases when the velocities of the reference and experimental media are extremely dissimilar, then the maximum error in group velocity can be greater than 2%. Depending on the particular




(a)

(b)

Fig. 5. (a) Comparison of the simulated diffraction corrupted velocity and experimental ultrasonic group velocity estimate for the 1 MHz transducer pair, versus distance. (b) Comparison of the simulated diffraction corrupted velocity and experimental ultrasonic group velocity estimate for the 500 kHz transducer pair, versus distance.




(a)

(b)

Fig. 6. (a) Corrected (GVDC) and uncorrected experimental ultrasonic group velocity estimates for the 500 kHz transducer pair, versus distance. (b) Corrected (GVDC) and uncorrected experimental ultrasonic group velocity estimates for the 1 MHz transducer pair, versus distance.

application, this may or may not represent an important effect. We also presented a technique, termed GVDC, for correcting the group velocity estimate. The technique was able to improve the accuracy of the velocity estimates by more than 40%.

Our simulations showed that the relative diffraction error depends on the difference between the velocities in the reference and sample media. For materials such as biological soft tissue measured in a reference medium of water, the diffraction errors would be almost nonexistent (see Fig. 7(b)). On the other hand, for materials with much larger acoustic velocities, such as bone, the diffraction effect might be important when the measurement is carried out in a water medium. In particular, recent studies on bone using insertion methods have not considered the effect of diffraction [26], [27]. Diffraction correction could improve the accuracy of the velocity estimates in such applications. With respect to clinical practice, the application of this technique would not be expected to lead to meaningful improvements in standard imaging modalities, at least with respect to soft tissue imaging. In future applications which may involve bone, however, this technique may eventually be useful as well.

We also note that we have assumed that the acoustic velocity within the sample medium is known *a priori* in order to apply our diffraction correction techniques. Of course, this will not be so in practice. However, since we have shown that the maximum error for the range of sample velocities presented, i.e., 1000 $ms^{-1}$–6000 $ms^{-1}$, is no more than 2.5%,

TABLE I
THE EXPERIMENTAL AVERAGE AND MAXIMUM ABSOLUTE PERCENT VELOCITY ERRORS AND RMS ERRORS DUE TO THE EFFECTS OF DIFFRACTION FOR THE 1 MHz TRANSDUCER PAIR

| | Empirical (Uncorrected) GV Estimate | GVDC Estimate | Percent Improvement |
|---|---|---|---|
| Avg. Abs. % Error | 0.15 | 0.075 | 50 |
| Max Abs. % Error | 0.40 | 0.26 | 35 |
| RMS | 5.10 | 2.50 | 51 |

TABLE II
THE EXPERIMENTAL AVERAGE AND MAXIMUM ABSOLUTE PRESENT VELOCITY ERRORS AND RMS ERRORS DUE TO THE EFFECTS OF DIFFRACTION FOR THE 500 kHz TRANSDUCER PAIR

| | Empirical (Uncorrected) GV Estimate | GVDC Estimate | Percent Improvement |
|---|---|---|---|
| Avg. Abs. % Error | 0.19 | 0.13 | 32 |
| Max Abs. % Error | 1.20 | 0.58 | 52 |
| RMS | 9.10 | 4.80 | 47 |

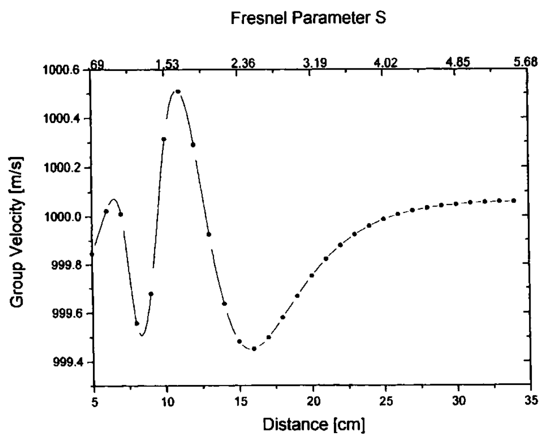
TABLE III
THE COMPUTER SIMULATED AVERAGE AND MAXIMUM ABSOLUTE PERCENT VELOCITY ERRORS DUE TO THE EFFECTS OF DIFFRACTION, FOR VARIOUS ULTRASONIC VELOCITIES ASSOCIATED WITH THE SAMPLE

| Vs [ m/s ] | Average Absolute Error % | Max Absolute Error % |
|---|---|---|
| 1000 | 0.02 | 0.06 |
| 1600 | 0.01 | 0.02 |
| 2770 | 0.13 | 0.37 |
| 6000 | 0.65 | 2.28 |

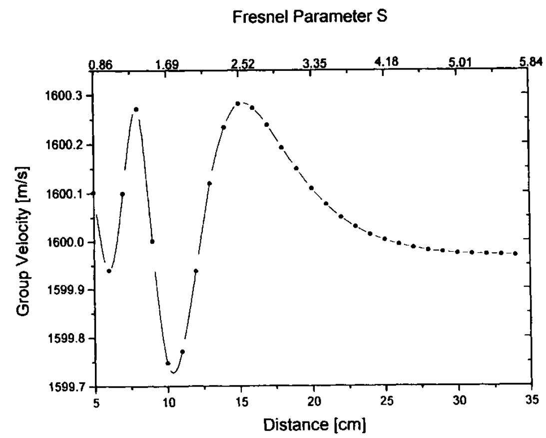
a first estimate of velocity can be obtained through direct measurement without any diffraction correction procedure. Once this initial estimate of velocity is obtained, it may be used in the GVDC scheme to increase the accuracy of the empirical group velocity estimate. In order to validate this approach, we carried out a simulation to characterize the error in the diffraction estimate when an incorrect value of velocity, $v_{s^*}$, was used in the diffraction transfer function. Fig. 8 presents the diffraction corrected group velocity estimate as a function of the value of velocity used in the diffraction transfer function numerical simulation. As may be seen, for a total change of 10% in the velocity used in the diffraction transfer function numerical evaluation, there is only a change of 0.02% in the diffraction corrected group velocity. Thus, small errors in the sample's acoustic velocity do not appear to be important in the diffraction correction procedure.

In addition, note that diffraction correction is not necessary when measurements can be made with transducers separated by sufficiently large distances. However, for practical reasons, this may not always be possible [26], [27]. Moreover, our analysis can be used to determine the minimum distance necessary for avoiding diffraction effects. Our results show that diffraction can be important for relatively large separation distances. For example, we observed significant errors in group velocity estimates for values of the Fresnel parameter $S > 3$. Note also that the diffraction correction technique as developed here does not require that transducers with identical diameters be used. The method can be applied in this case by changing the integration limits in (14) and recomputing the diffraction transfer function. It thus could be used to evaluate the effects on accuracy of using a hydrophone receiver. For example, the effects of coaxially misaligning the receiver with respect to the transmitter axis could be directly evaluated.

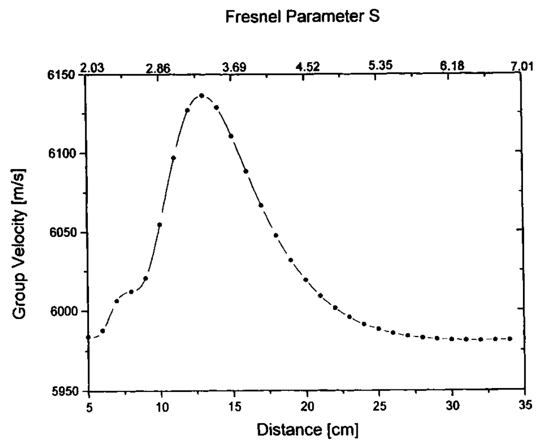
We should also note that we have only addressed the issue of systematic errors introduced by diffraction. The issue of precision has not been discussed. It seems reasonable to assume that the precision of the group velocity estimates may also be affected by the diffraction phenomena through changes in the noise statistics. Future studies may provide further information on this subject.




(a)




(b)




(c)

Fig. 7. (a) Simulated diffraction corrupted ultrasonic group velocity estimate versus distance, for an actual acoustic velocity in the sample of 1000 $ms^{-1}$. (b) Simulated diffraction corrupted ultrasonic group velocity estimate versus distance, for an actual acoustic velocity in the sample of 1600 $ms^{-1}$. (c) Simulated diffraction corrupted ultrasonic group velocity estimate versus distance, for an actual acoustic velocity in the sample of 6000 $ms^{-1}$.

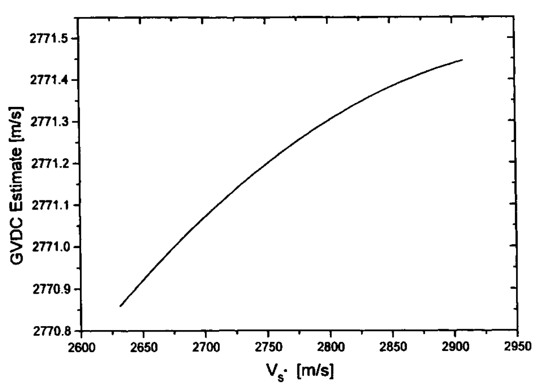



Fig. 8. Diffraction corrected group velocity estimate versus $v_{s^*}$. The velocity, $v_{s^*}$, is used in the diffraction transfer function to correct the group velocity estimate (see (10)).

In conclusion, diffraction effects can cause small but potentially significant errors in estimates of group velocity with insertion measurements. The diffraction correction technique presented here should enable more accurate estimates of group velocity to be made, if required. It is relatively easy to implement and its software requirements relatively modest. Most importantly, the approach described here can serve as the means by which to assess how diffraction can affect ultrasonic group velocity estimates in various types of insertion mode experiments.

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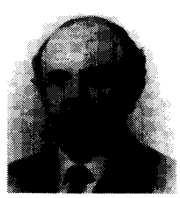

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